Technical Comments

Comment on "Base Pressure Measurements on Sharp and Blunt 9° Cones at Mach Numbers from 3.50 to 9.20"

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ZARIN[1] concludes that the semiempirical estimates given by the present authors in Ref. 2 may lead to erroneous conclusions if used above Mach 4.5 and, therefore, should be avoided. Zarin is apparently confused concerning the applicable conditions for the high Mach number blunt cone estimates given in Ref. 2. Zarin's comparison of his new experimental base pressure data for a blunt 9° cone and the estimate based on Ref. 2 is shown in Fig. 1. The estimate of Ref. 2 shown here is for the essentially limiting or lower base pressure corresponding to fully turbulent boundary-layer flow (Reynolds number $\gtrsim 40 \times 10^6$). As noted in Fig. 1, Zarin's data are as much as three orders in Reynolds number below the applicable conditions. Although not discussed in Ref. 1, it should also be noted that Zarin reported in Ref. 3 that his highest Mach number (9.2) data were obtained with a laminar boundary-layer flow, and his Mach 5 and 7.5 data correspond to transitional flow. Even a successfully tripped boundary layer will not, of course, produce data directly comparable to the estimate of Ref. 2 since the relative boundary-layer thickness still would be too large.

Actually, the experimental data of Ref. 1 strongly support the analysis given in Ref. 2. The present authors predicted, at a time when systematic high Mach number base pressure data were not available, that specific, blunt vehicles would encounter a minimum and then a subsequent increase in base pressure with increasing flight Mach number. Zarin's data, although too many decades low in Reynolds number for quantitative comparison, clearly demonstrate this phenomena.

Zarin proposes in Ref. 1 a new empirical correlation of base pressure for both blunt and sharp cones. Although he points out that his data indicate more influence of bluntness than Reynolds number, he proposes a correlation based only on Reynolds number and states that its simplicity will justify its usefulness. The latter is a weak excuse for the liberties taken in order to show a set of linear curves connecting selected groups of data points representing blunt cones at the low Re_l extremes and sharp cones at the high Re_l extremes, without regard for the fact that substantial differences in local Mach numbers exist on these sharp and blunt cones. The obviously unjustified demand for a linear variation of P_b with log Re_l at constant M_∞ leads to disregard of large percentage discrepancies between the alleged correlation and Zarin's own data. Close examination of Fig. 1 in Ref. 1 will reveal that Zarin's simple correlation yields negative absolute pressures even within the Reynolds number range of his own

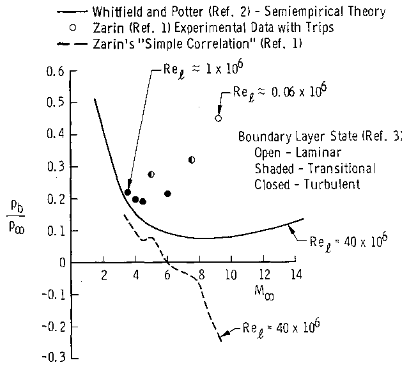

Fig. 1 Base pressure ratios for blunt 9° cone.

data. Zarin presented his correlation in terms of the base pressure coefficient,

$$P_b = (p_b - p_\infty)/\tfrac{1}{2}\rho_\infty u_\infty^2$$

and apparently overlooked the fact that $P_b = -0.01688$ corresponds to zero absolute base pressure at $M_\infty = 9.2$ and $\gamma_\infty = 1.4$. Zarin's "simple correlation" is compared with the estimate of Ref. 2 in Fig. 1. This shows that the correlation has no meaning for high Mach number and high Reynolds number flows.

References

[1] Zarin, N. A., "Base pressure measurements on sharp and blunt 9° cones at Mach numbers from 3.50 to 9.20," AIAA J. 4, 743–745 (1966).

[2] Whitfield, J. D. and Potter, J. L., "On base pressures at high Reynolds numbers and hypersonic Mach numbers," Arnold Engineering Development Center, AEDC-TN-60-61 (March 1960).

[3] Zarin, N. A., "Base pressure measurements on sharp and blunt 9° cones at Mach numbers from 3.50 to 9.20," Ballistic Research Labs. Memo. Rept. 1709 (November 1965).

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Reply by Author to J. D. Whitfield and J. L. Potter

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THE author wishes to thank Messers. Whitfield and Potter for their preceding comment, parts of which are valid, and parts of which need some further clarification. Concerning the applicable conditions for the high Mach number blunt cone estimates given in Ref. 1, it was stated by Whitfield and Potter that the Reynolds number of 40×10^6 was taken as being that Reynolds number where p_b/p_1 becomes almost constant. The blunt cone data that they used in obtaining the curves in Fig. 8 of Ref. 1 were obtained at a Reynolds number of less than half that value, although their criterion of having a nearly constant value of base pressure ratio with re-

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spect to Reynolds number was satisfied. In keeping with this criterion, this author used his blunt cone data where p_b/p_1 had reached a constant or nearly constant value, even though the data was obtained at Reynolds numbers less than 40×10^6.

The data points for Mach 7.5 and 9.2 shown in Fig. 1 of the preceding comment were for transitional and laminar flow, respectively, and should not have appeared on the graph. The data point at Mach 5.0 was for turbulent flow, but was not included in the similar figures in this author's original report and article[2,3] since the value of the base pressure ratio had not reached a constant value.

It was shown by this author[2] that once turbulent flow was obtained over the model, the base pressure showed very little dependence on how transition was achieved (naturally or with trip ring).

With regard to the empirical correlation proposed in Refs. 2 and 3, this author is grateful to Messers. Whitfield and Potter for pointing out its anomalous behavior at high Mach numbers and Reynolds numbers. The temptation to draw straight lines through data points is a strong one, and, in this case, such an action was clearly ill advised.

References

[1] Whitfield, J. D. and Potter, J. L., "On base pressures at high Reynolds numbers and hypersonic Mach numbers," Arnold Engineering Development Center, AEDC-TN-60-61 (March 1960).

[2] Zarin, N. A., "Base pressure measurements on sharp and blunt 9° cones at Mach numbers from 3.50 to 9.20," Ballistic Research Labs. Memo. Rept. 1709 (November 1965).

[3] Zarin, N. A., "Base pressure measurements on sharp and blunt 9° cones at Mach numbers from 3.50 to 9.20," AIAA J. 4, 743–745 (1966).

Effect of Modes on Plastic Buckling of Compressed Cylindrical Shells

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Introduction

THE fundamental case of plastic buckling of axially compressed cylindrical shells has been the subject of intensive research during past years.[1–4] However, some aspects of the problem are not completely resolved, and considerable controversy still wages over basic issues.

Much of the controversy centers around the use of deformation vs incremental theories of plasticity in buckling problems.[2] Briefly, it is well known that to describe time-independent irreversible material behavior in a mathematically and physically rigorous fashion, an incremental theory of plasticity must be used. However, this does not rule out the use of deformation theories (which in general are not rigorous descriptions of material behavior) in some problems if certain real restrictions on the loading surface and loading path are met.[5] Three remarks are in order in this latter connection: 1) deformation theory is equivalent to an integrated incremental theory, and hence a rigorous description of material behavior, for the case of proportional loading, 2) the allowable deviations from proportional loading[5] are not as restrictive as the necessary introduction of a corner appearing in the loading surface in a very special way, and 3) the restrictions on the application of deformation theory in buckling problems have not, to date, been satisfied.

A supposed paradox in plastic buckling problems is that predictions of buckling stresses using an incremental theory often bear little resemblance to experimentally obtained loads, whereas deformation theory predictions are excellent in general. Onat and Drucker[6] examined this paradox for the compressed cruciform, or the equivalent plate problem, which fails by twisting. They showed that small and therefore unavoidable imperfections in shape do account for the paradox. Furthermore, the effect of initial imperfections was very large for incremental theory, but it was relatively small for deformation theory.

Lee[1] examined the situation for axially compressed, initially imperfect, cylindrical shells. A diamond-shaped circumferential wave mode of initial imperfection and subsequent deflection was assumed. The results of Lee[1] appeared to show that incremental theory, even with initial imperfections, overestimated the buckling strength whereas deformation theory predicted it quite well. In addition, the effect of imperfections was the same order of magnitude for both incremental and deformation theory.

Recently Batterman[2] reexamined the fundamental cylindrical shell case and showed that in the range of cylinders tested by Lee, an axisymmetric mode of buckling should have been used to bring incremental theory into better agreement with tests. The results of Batterman were for perfect cylinders where due attention was paid to unloading.

The main purpose of this note is to correct an error in Ref. 1 which further explains some anomalies in the cylindrical shell buckling phenomenon. It will be shown that the mode of buckling is crucial for the success of incremental theory but is not as vital for deformation theory. Furthermore, the effect of imperfections for the circumferential mode will be seen to be extremely large for incremental theory in contrast to the results of Lee,[1] and in agreement with Onat and Drucker.[6]

Analysis

Consider a perfect cylindrical shell of radius R and thickness t which is under uniaxial compression in the plastic range prior to buckling. If buckling occurs in the diamond-shaped circumferential mode with no unloading, then for J_2 incremental theory the critical stress is[1]

$$\sigma_{cr}^{CM} = \frac{2}{3} E \frac{t}{R} \left\{ \frac{3}{(5-4\nu)\lambda - (1-2\nu)^2} \times \left[\frac{[\lambda(\lambda+3)]^{1/2} + \dfrac{(7-2\nu)\lambda + 3(2\nu-1)}{2(1+\nu)}}{[\lambda(\lambda+3)]^{1/2} + 3 - \lambda} \right] \right\}^{1/2} \quad (1)$$

where E is the modulus of elasticity, ν is Poisson's ratio, and $\lambda = E/E_T$ where E_T is the tangent modulus at buckling. Note that for $\lambda = 1$, (1) reduces to the elastic formula

$$\sigma_{cr} = E(t/R)\{1/[3(1-\nu^2)]^{1/2}\} \quad (2)$$

If we consider the same problem for axisymmetric buckling, the critial stress at which buckling is first possible is [2,7]

$$\sigma_{cr}^{AX} = \tfrac{2}{3}E(t/R)[3/\{(5-4\nu)\lambda - (1-2\nu)^2\}]^{1/2} \quad (3)$$

For elastic buckling ($\lambda = 1$), (3) reduces to (2) as it should. However, note that for plastic buckling, ($\lambda > 1$) σ_{cr}^{AX} will always be less than σ_{cr}^{CM}, a fact first reported by Bijlaard.[8]

It is important to realize (and previously unreported) that as buckling occurs at smaller values of the tangent modulus,

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